

# Higgs Potential in Minimal $S_3$ Invariant Extension of the Standard Model

Jisuke Kubo<sup>1,2</sup>, Hiroshi Okada<sup>2</sup>, and Fumiaki Sakamaki<sup>2</sup>

<sup>1</sup> *Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, D-80805 Munich, Germany*

<sup>2</sup> *Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan*

## Abstract

Minimal  $S_3$  invariant Higgs potential with real soft  $S_3$  breaking masses is investigated. It is required that without having a problem with triviality, all physical Higgs bosons, except one neutral one, become heavy  $\gtrsim 10$  TeV in order to sufficiently suppress flavor changing neutral currents. There exist three nonequivalent soft mass terms that can be characterized according to their discrete symmetries, and the one which breaks  $S_3$  completely. The  $S'_2$  invariant vacuum expectation values (VEVs) of the Higgs fields are the most economic VEVs in the sense that the freedom of VEVs can be completely absorbed into the Yukawa couplings so that it is possible to derive, without referring to the details of the VEVs, the most general form for the fermion mass matrices in minimal  $S_3$  extension of the standard model. We find that except for the completely broken case of the soft terms, the  $S'_2$  invariant VEVs are unique VEVs that satisfy the requirement of heavy Higgs bosons. It is found that they also correspond to a local minimum in the completely broken case.

PACS numbers: 11.30.Hv, 12.60.Fr, 12.15.Ff

## I. INTRODUCTION

A nonabelian flavor symmetry is certainly a powerful tool to understand flavor physics. In the case of the standard model (SM), where only one Higgs  $SU(2)_L$  doublet is present, any nonabelian flavor symmetry has to be explicitly broken to describe experimental data. However, if the Higgs sector is extended, and Higgs fields belong to a nontrivial representation of a flavor group [1, 2], phenomenologically viable possibilities may arise. The smallest nonabelian discrete group is  $S_3$ <sup>1</sup>. It is a permutation group of three objects, and offers a possible explanation why there are three generations of the quarks and leptons [8, 9]. An  $S_3$  invariant Yukawa sector of the SM has exactly five independent couplings [8, 9]

$$\begin{aligned}
1 &: L_a R_a H_a + L_b R_b H_b + L_c R_c H_c, \\
2 &: L_a (R_b + R_c) H_a + L_b (R_a + R_c) H_b + L_c (R_a + R_b) H_c, \\
3 &: (L_b + L_c) R_a H_a + (L_a + L_c) R_b H_b + (L_a + L_b) R_c H_c, \\
4 &: (L_b R_b + L_c R_c) H_a + (L_a R_a + L_c R_c) H_b + (L_a R_a + L_b R_b) H_c, \\
5 &: (L_b R_c + L_c R_b) H_a + (L_a R_c + L_c R_a) H_b + (L_a R_b + L_b R_a) H_c,
\end{aligned} \tag{1}$$

where  $L_a$ ,  $R_a$  and  $H_a$  correspond to three left-handed leptons, right-handed leptons and Higgs bosons, which are subject to permutations. The three dimensional representation  $\mathbf{3}$  of  $S_3$  is not an irreducible representation;  $\mathbf{3}$  can be decomposed into  $\mathbf{1}$  and  $\mathbf{2}$  as

$$\mathbf{1} : H_S = \frac{1}{\sqrt{3}}(H_a + H_b + H_c), \tag{2}$$

$$\mathbf{2} : (H_1, H_2) = \left( \frac{1}{\sqrt{2}}(H_a - H_b), \frac{1}{\sqrt{6}}(H_a + H_b - 2H_c) \right), \tag{3}$$

and similarly for  $L$ 's and  $R$ 's. In terms of the fields in the irreducible basis, the five independent Yukawa couplings are [8, 9]:

$$L_i R_i H_S, \quad f_{ijk} L_i R_j H_k, \quad L_S R_S H_S, \quad L_S R_i H_i, \quad L_i R_S H_i, \tag{4}$$

---

<sup>1</sup> Flavor symmetries based on a permutation symmetry have been considered by many authors in the past. One of the first papers on permutation symmetries are [1, 2, 3, 5, 6]. See [7] for a review. Phenomenologically viable models based on nonabelian discrete flavor symmetries  $S_3$ ,  $D_4$  and  $A_4$  and also on a product of abelian discrete symmetries have been recently constructed in [8, 9, 10, 11, 12], [14], [15, 16, 17, 18] and [19, 20], respectively. (See also [21, 22, 23, 24, 25].) However, it is difficult to understand bi-large mixing of neutrinos in terms of abelian discrete symmetries alone [13].

where  $i, j, k$  run from 1 to 2, and

$$f_{112} = f_{121} = f_{211} = -f_{222} = 1. \quad (5)$$

It has been found in [8, 9] that these Yukawa couplings are sufficient to reproduce the masses of the quarks and their mixing, and that they are not only consistent with the known observations in the leptonic sector, but also can make testable predictions in the neutrino sector if one assumes an additional discrete symmetry in this sector. In deriving the fermion mass matrices, it has been assumed in [8, 9] that the vacuum expectation values (VEVs) of the Higgs fields are  $S'_2$  invariant, i.e.

$$\langle H_S \rangle \neq 0, \quad \langle H_1 \rangle = \langle H_2 \rangle \neq 0. \quad (6)$$

By the  $S'_2$  invariance we mean an invariance under the interchange of  $H_1$  and  $H_2$ , i.e.

$$H_1 \leftrightarrow H_2. \quad (7)$$

Note that this permutation symmetry is not a subgroup of the original  $S_3$ . Although the Yukawa couplings (4) do not respect this symmetry, each term in the  $S_3$  invariant Higgs potential (given in (9)) except for one term respects this discrete symmetry. Moreover, as we can see from (4), the  $S'_2$  invariant VEVs (6) are the most economic VEVs in the sense that the freedom of VEVs can be completely absorbed into the Yukawa couplings so that we can derive the most general form for the fermion mass matrices

$$\mathbf{M} = \begin{pmatrix} m_1 + m_2 & m_2 & m_5 \\ m_2 & m_1 - m_2 & m_5 \\ m_4 & m_4 & m_3 \end{pmatrix} \quad (8)$$

without referring to the details of VEVs. In other words, if  $\langle H_1 \rangle \neq \langle H_2 \rangle$ , the mass matrices would have one more independent parameter which should be determined in the Higgs sector.

In the present paper we would like to investigate how unique the  $S'_2$  invariant vacuum is under the requirement that except one neutral physical Higgs boson all the physical Higgs bosons can become heavy  $\gtrsim 10$  TeV without having a problem with triviality [26]. This bound results in order to suppress three-level flavor changing neutral currents (FCNCs) that contribute, for instance, to the mass difference  $\Delta m_K$  of  $K^0$  and  $\bar{K}^0$  in  $S_3$  invariant

extension of the SM [27, 28]. [See also Ref. [3] and [4].] The investigations are presented in Sect. 3 and 4, and the conclusions are summarized in the last section. In Sect. V we discuss the Pakvasa-Sugawara vacuum [1], and a supersymmetric case is treated in Sect. 6.

## II. $S_3$ INVARIANT HIGGS POTENTIAL AND SOFT $S_3$ BREAKING

### A. $S_3$ invariant Higgs potential and its problem

The most general,  $S_3$  invariant, renormalizable potential is given by [1]

$$V_H = V_{2H} + V_{4H}, \quad (9)$$

$$V_{2H} = -\mu_1^2(H_1^\dagger H_1 + H_2^\dagger H_2) - \mu_3^2 H_S^\dagger H_S,$$

$$\begin{aligned} V_{4H} = & +\lambda_1(H_1^\dagger H_1 + H_2^\dagger H_2)^2 + \lambda_2(H_1^\dagger H_2 - H_2^\dagger H_1)^2 \\ & +\lambda_3((H_1^\dagger H_2 + H_2^\dagger H_1)^2 + (H_1^\dagger H_1 - H_2^\dagger H_2)^2) \\ & +\{\lambda_4 f_{ijk}(H_S^\dagger H_i)(H_j^\dagger H_k) + h.c.\} + \lambda_5(H_S^\dagger H_S)(H_1^\dagger H_1 + H_2^\dagger H_2) \\ & +\lambda_6\{(H_S^\dagger H_1)(H_1^\dagger H_S) + (H_S^\dagger H_2)(H_2^\dagger H_S)\} \\ & +\{\lambda_7[(H_S^\dagger H_1)(H_S^\dagger H_1) + (H_S^\dagger H_2)(H_S^\dagger H_2)] + h.c.\} \\ & +\lambda_8(H_S^\dagger H_S)^2, \end{aligned} \quad (10)$$

where  $\lambda_4$  and  $\lambda_7$  can be complex <sup>2</sup>. We first redefine  $H_i$  as

$$H_\pm = \frac{1}{\sqrt{2}}(H_1 \pm H_2), \quad (11)$$

and write the  $SU(2)_L$  Higgs doublets in components:

$$H_\pm = \begin{pmatrix} h_\pm + i\chi_\pm \\ \frac{1}{\sqrt{2}}(h_\pm^0 + i\chi_\pm^0) \end{pmatrix}, \quad H_S = \begin{pmatrix} h_S + i\chi_S \\ \frac{1}{\sqrt{2}}(h_S^0 + i\chi_S^0) \end{pmatrix}. \quad (12)$$

The down components of the Higgs doublets have zero electric charge, and therefore, we assume that only the down components can acquire a VEV. Further, because of  $U(1)_Y$  gauge invariance, it is always possible to make a phase rotation for  $H_S$  so that only the real part  $h_S^0$  can get VEV. We denote the VEVs as follows:

$$\langle h_\pm^0 \rangle = v_\pm, \quad \langle h_S^0 \rangle = v_S, \quad \langle \chi_\pm \rangle = c_\pm, \quad (13)$$

---

<sup>2</sup> The  $S_3$  invariant potential has been studied in [1, 21], for instance. Similar potentials with nonabelian discrete symmetries have been also studied in [2, 3, 14, 29].

which should satisfy the constraint

$$(v_+^2 + v_-^2 + v_S^2 + c_+^2 + c_-^2)^{1/2} = v \simeq 246 \text{ GeV}. \quad (14)$$

In order to reproduce realistic fermion masses and their mixings [8], we also require that

$$v_S \neq 0, \text{ and at least one of } v_{\pm} \text{ and } c_{\pm} \neq 0 \quad (15)$$

is satisfied, and do not allow a large hierarchy among the nonvanishing VEVs, unless it is noticed. [In sections II and III, we however allow such hierarchy.]

There are five minimization conditions:

$$0 = -v_S \mu_3^2 + \partial V_{4H} / \partial h_S^0, \quad (16)$$

$$0 = -v_+ \mu_1^2 + \partial V_{4H} / \partial h_+^0, \quad (17)$$

$$0 = -v_- \mu_1^2 + \partial V_{4H} / \partial h_-^0, \quad (18)$$

$$0 = -c_+ \mu_1^2 + \partial V_{4H} / \partial \chi_+^0, \quad (19)$$

$$0 = -c_- \mu_1^2 + \partial V_{4H} / \partial \chi_-^0, \quad (20)$$

We regard VEVs as independent parameters, and express the parameters of the potential (9), especially the mass parameters  $\mu_1^2$  and  $\mu_3^2$ , in terms of the VEVs. To make all the physical Higgs bosons except one neutral Higgs boson without having large values of the Higgs quartic couplings  $\lambda$ 's, we have to have either  $-\mu_3^2, -\mu_1^2 \gg v^2$  or  $-\mu_1^2 \gg v^2$ , where  $v$  is defined in (14). For the first case, none of the VEVs can be  $O(v)$ , because the derivative terms, i.e.  $\partial V_{4H} / \partial h_+^0$  etc, are of  $O(\text{VEV}^3)$ . Therefore, this case can not satisfy the constraint (14). For the second case,  $\mu_3$  and  $v_S$  can be  $O(v)$ , but none of  $v_+, v_-, c_+, c_-$  can be  $O(v)$ . That is, the hierarchy  $|v_+/v_S|, |v_-/v_S|, |c_+/v_S|, |c_-/v_S| \ll 1$  has to be satisfied. This hierarchy is consistent with the minimization conditions (17)–(20), only if at least one of the derivative terms, i.e.  $\partial V_{4H} / \partial h_+^0$  etc, contains at least a term proportional to  $v_S^3$ . However, this is not the case, as we can see from the potential  $V_{4H}$  (10). Moreover, (15) does not allow  $v_+ = v_- = c_+ = c_- = 0$ .

It is thus clear, if the two conditions (14) and (15) are satisfied, that  $\mu_1^2, \mu_3^2 \sim O(\text{VEV}^2)$ , which means that all the masses of the physical Higgs bosons are of  $O(\text{VEV})$ . That is, to have a large Higgs mass, the value of certain Higgs couplings  $\lambda$ 's have to be large. Then we are running into the problem with triviality; the Higgs mass can not be larger than

the cutoff. As we see from (9), the model has many Higgs couplings, so that the known triviality bound on the Higgs mass,  $\sim 700$  GeV [26], can not be directly applied. But we may assume that the bound for the present case does not differ very much from that of the SM. However, this upper bound is too low to suppress three-level flavor changing neutral currents (FCNCs) that contribute, for instance, to the mass difference  $\Delta m_K$  of  $K^0$  and  $\bar{K}^0$ ; certain Higgs masses in  $S_3$  invariant extension of the SM have to be larger than  $\sim O(10)$  TeV [3, 27, 28]. Therefore, in a phenomenologically viable  $S_3$  extension of the SM,  $S_3$  symmetry should be broken, unless there is some cancellation mechanism of FCNCs.

### B. Soft $S_3$ breaking terms and their characterization

As we have seen above, we have to modify the Higgs potential (9) to make it possible that the Higgs masses can become larger than 10 TeV. How should we break  $S_3$ ? We would like to maintain the consistency and predictions of  $S_3$  in the Yukawa sector, while satisfying simultaneously the experimental constraints from FCNC phenomena. Therefore, we break  $S_3$  as soft as possible. The softest operators in the case at hand are those of dimension two; that is, mass terms. There are four soft breaking mass terms

$$\begin{aligned} V_{SB} = & -\mu_2^2(H_+^\dagger H_+ - H_-^\dagger H_-) - \sqrt{2}(\mu_4^2 H_S^\dagger H_+ + h.c.) \\ & -(\mu_5^2 H_+^\dagger H_- + h.c.) - \sqrt{2}(\mu_6^2 H_S^\dagger H_- + h.c.). \end{aligned} \quad (21)$$

$\mu_4^2, \mu_5^2$  and  $\mu_6^2$  can be complex parameters<sup>3</sup>. However, we assume that they are real parameters in following discussions except in Sect. 5. We would like to characterize these four mass terms according to discrete symmetries:

$$R : H_S \rightarrow -H_S, \quad (22)$$

$$S'_2 : H_- \rightarrow -H_-, \quad (23)$$

$$S''_2 : H_+ \rightarrow -H_+, \quad (24)$$

$$R \times S'_2 : H_S \rightarrow -H_S \text{ and } H_- \rightarrow -H_-, \quad (25)$$

$$R \times S''_2 : H_S \rightarrow -H_S \text{ and } H_+ \rightarrow -H_+ \quad (26)$$

$$S'_2 \times S''_2 : H_- \rightarrow -H_- \text{ and } H_+ \rightarrow -H_+, \quad (27)$$

---

<sup>3</sup> The soft mass terms (21) may be generated from a  $S_3$  invariant Higgs potential by introducing certain  $S_3$  singlet Higgs fields [4].

where  $S'_2$  and  $S''_2$  are not a subgroup of the original  $S_3$ . Accordingly, we characterize the soft mass terms (21) as

$$R : \mu_4 = \mu_6 = 0, \quad (28)$$

$$S'_2 : \mu_5 = \mu_6 = 0, \quad (29)$$

$$S''_2 : \mu_4 = \mu_5 = 0, \quad (30)$$

$$R \times S'_2 : \mu_4 = \mu_5 = \mu_6 = 0, \quad (31)$$

$$R \times S''_2 : \mu_4 = \mu_5 = \mu_6 = 0, \quad (32)$$

$$S'_2 \times S''_2 : \mu_4 = \mu_5 = \mu_6 = 0. \quad (33)$$

Actually, there are only four nonequivalent soft breaking mass terms, including one without no discrete symmetry. This is because  $S'_2$  and  $S''_2$  are not independent: The Higgs potential (9) and the soft terms (21) are invariant under the interchange of  $H_+$  and  $H_-$  if one appropriately redefines the coupling constants and mass parameters. In the next section we will discuss the three cases, i.e.  $R$ ,  $S'_2$  and  $R \times S'_2$  invariant cases, and in Sect. 4 we will treat the completely broken case, in which all the soft mass terms (21) are present. Each possibility is renormalizable, because all the other interactions are  $S_3$  invariant and can not induce infinite  $S_3$  violating breaking terms (21). In principle,  $\mu_4^2, \mu_5^2$  and  $\mu_6$  can be complex. As announced, however, we assume that they are real, except for Sect. 5. This is consistent with renormalizability from the same reason above.

Before we go to the next sections, it may be worthwhile to write down explicitly the  $\lambda_4$  and  $\lambda_7$  terms of the potential  $V_{4H}$  (10):

$$\begin{aligned} 2\sqrt{2}V_{\lambda_4 H} = & (\text{Re}(\lambda_4)\chi_S^0 - \text{Im}(\lambda_4)h_S^0) [(\chi_+^0)^3 + 3(\chi_+^0)^2\chi_-^0 - 3\chi_+^0(\chi_-^0)^2 - (\chi_-^0)^3 \\ & + \chi_+^0(h_+^0)^2 + \chi_-^0(h_+^0)^2 + 2\chi_+^0h_+^0h_-^0 - 2\chi_-^0h_+^0h_-^0 - \chi_+^0(h_-^0)^2 - \chi_-^0(h_-^0)^2] \\ & + (\text{Re}(\lambda_4)h_S^0 + \text{Im}(\lambda_4)\chi_S^0) [(\chi_+^0)^2h_+^0 - (\chi_-^0)^2h_+^0 + 2\chi_+^0\chi_-^0h_+^0 - 2\chi_+^0\chi_-^0h_-^0 \\ & + (h_+^0)^3 - (h_-^0)^3 + (\chi_+^0)^2h_-^0 - (\chi_-^0)^2h_-^0 + 3(h_+^0)^2h_-^0 - 3h_+^0(h_-^0)^2], \end{aligned} \quad (34)$$

$$\begin{aligned} V_{\lambda_7 H} = & \frac{\text{Re}(\lambda_7)}{2} [\{(\chi_+^0)^2 + (\chi_-^0)^2 - (h_+^0)^2 - (h_-^0)^2\}\{(\chi_S^0)^2 - (h_S^0)^2\} \\ & + 4\{\chi_+^0h_+^0 + \chi_-^0h_-^0\}\chi_S^0h_S^0] + \text{Im}(\lambda_7) [(\chi_+^0h_+^0 + \chi_-^0h_-^0)\{(\chi_S^0)^2 - (h_S^0)^2\} \\ & - \{(\chi_+^0)^2 + (\chi_-^0)^2 - (h_+^0)^2 - (h_-^0)^2\}\chi_S^0h_S^0], \end{aligned} \quad (35)$$

where only those terms containing the neutral components are written above. The rest of

the terms in  $V_{4H}$  has the form

$$(h_S^0)^{2n_1}(h_+^0)^{2n_2}(h_-^0)^{2n_3}(\chi_S^0)^{2n_4}(\chi_+^0)^{2n_5}(\chi_-^0)^{2n_6} \quad (36)$$

with  $\sum_{i=1}^6 n_i = 2$  and  $n_i = 0, 1, 2$ .

### III. MINIMIZATION CONDITIONS AND HIGGS MASSES

Below we will analyze the total potential  $V_T = V_H + V_{SB}$  for the three nonequivalent cases (28), (29) and (31). We consider only phenomenologically viable cases (15). But we do allow if necessary a large hierarchy among the nonvanishing VEVs. In all the cases,  $\lambda_4 = 0$  follows from the discrete symmetry in question.

$R \times S'_2$  ( $\mu_4 = \mu_5 = \mu_6 = 0; \lambda_4 = 0$ ):

The five minimization conditions in this case are given by

$$0 = -v_S \mu_3^2 + \partial V_{4H} / \partial h_S^0, \quad (37)$$

$$0 = -v_+(\mu_1^2 + \mu_2^2) + \partial V_{4H} / \partial h_+^0, \quad (38)$$

$$0 = -v_-(\mu_1^2 - \mu_2^2) + \partial V_{4H} / \partial h_-^0, \quad (39)$$

$$0 = -c_+(\mu_1^2 + \mu_2^2) + \partial V_{4H} / \partial \chi_+^0, \quad (40)$$

$$0 = -c_-(\mu_1^2 - \mu_2^2) + \partial V_{4H} / \partial \chi_-^0, \quad (41)$$

where the second derivative terms, i.e.,  $\partial V_{4H} / \partial h^0$  and  $\partial V_{4H} / \partial \chi^0$ , are  $\sim O(\text{VEV}^3)$ . We first observe that because of the absence of  $\lambda_4$  the condition (37) requires  $\mu_3 \sim O(\text{VEV})$ . If  $|\mu_1^2 \pm \mu_2^2| \gg v^2$  should be satisfied, then none of  $v_+, v_-, c_+, c_-$  can be  $O(v)$ . But this is not consistent with (38)–(41) because of the absence of  $v_S^3$  terms in the derivative terms of (38)–(41). Therefore, taking into account the condition (15), at least one of  $v_+, v_-, c_+, c_-$  has to be  $O(v)$ . Assume that  $v_+ \sim O(v)$ , which means that  $\mu_1^2 = -\mu_2^2 + O(\text{VEV}^2)$ . Consequently, the total Higgs potential in this case can be written as

$$V_T = -2\mu_1^2 H_-^\dagger H_- + \dots, \quad (42)$$

where the terms indicated by  $\dots$  are those which are proportional to  $\text{VEV}^n$  ( $n = 1, \dots, 4$ ). Therefore, only  $H_-$  can obtain a large mass, if  $-2\mu_1^2$  is positive and large. So, this case does



not satisfy the phenomenological requirement that all the physical Higgs bosons except one can be made heavy without running into the problem with triviality.

One can perform similar analyses for other cases such as  $c_- \sim 0(v)$ . [ $v_S \neq 0$  is always assumed.] As before, one finds that only one  $SU(2)_L$  doublet can become heavy. So, the soft masses with the discrete symmetry  $R \times S'_2$  can not be used for a phenomenologically viable model.

R ( $\mu_4 = \mu_6 = 0; \lambda_4 = 0$ ):

The five minimization conditions in this case are given by

$$0 = -v_S \mu_3^2 + \partial V_{4H} / \partial h_S^0 \quad (43)$$

$$0 = -v_+(\mu_1^2 + \mu_2^2) - v_- \mu_5^2 + \partial V_{4H} / \partial h_+^0, \quad (44)$$

$$0 = -v_+ \mu_5^2 - v_-(\mu_1^2 - \mu_2^2) + \partial V_{4H} / \partial h_-^0, \quad (45)$$

$$0 = -c_+(\mu_1^2 + \mu_2^2) - c_- \mu_5^2 + \partial V_{4H} / \partial \chi_+^0, \quad (46)$$

$$0 = -c_+ \mu_5^2 - c_-(\mu_1^2 - \mu_2^2) + \partial V_{4H} / \partial \chi_-^0. \quad (47)$$

Again, because of (43),  $\mu_3 \sim O(\text{VEV})$ .  $|\mu_5|$  has to be large, otherwise the situation is the same as in the previous case. Eqs. (44) and (45) have a nontrivial solution

$$\mu_1^2 = -\frac{\mu_5^2(v_+^2 + v_-^2) + O(\text{VEV}^4)}{2v_+v_-}, \quad \mu_2^2 = \frac{\mu_5^2(v_+^2 - v_-^2) + O(\text{VEV}^4)}{2v_+v_-}, \quad (48)$$

if  $v_+ \neq 0, v_- \neq 0$ . Then the total potential becomes

$$V_T = m_H^2 H_H^\dagger H_H + \dots, \quad (49)$$

where, as before, the terms indicated by  $\dots$  are those which are proportional to  $\text{VEV}^n$  ( $n = 1, \dots, 4$ ), and

$$H_H = \frac{v_- H_+ - v_+ H_-}{(v_+^2 + v_-^2)^{1/2}}, \quad m_H^2 = \frac{v_+^2 + v_-^2}{v_+ v_-} \mu_5^2. \quad (50)$$

Therefore, only  $H_H$  can become heavy.

If  $v_- = 0$ , Eq. (45) requires  $|v_+/v| \ll 1$  because  $|\mu_5| \gg v$  has to be satisfied. To satisfy Eq. (45), on one hand, at least one of  $c_+$ , and  $c_-$  has to be  $O(v)$  because of the absence of  $v_S^3$  terms in the derivative term. On the other hand, we obtain the equation (48) with  $v_\pm \rightarrow c_\pm$ . [ $c_+ \sim O(\text{VEV}), c_- = 0$  and  $c_+ = 0, c_- = O(\text{VEV})$  can not satisfy (46) and (47).]

The case  $v_+ = 0$  is equivalent to the case  $v_- = 0$ . If  $v_+ = v_- = 0$ , the situation does not change. From these considerations, we conclude that the case at hand does not satisfy the phenomenological requirement.

$S'_2$ : ( $\mu_5 = \mu_6 = 0$ ;  $\lambda_4 = 0$ ):

The five minimization conditions in this case are given by

$$0 = -v_S \mu_3^2 - \sqrt{2} v_+ \mu_4^2 + \partial V_{4H} / \partial h_S^0, \quad (51)$$

$$0 = -v_+ (\mu_1^2 + \mu_2^2) - \sqrt{2} v_S \mu_4^2 + \partial V_{4H} / \partial h_+^0, \quad (52)$$

$$0 = -v_- (\mu_1^2 - \mu_2^2) + \partial V_{4H} / \partial h_-^0, \quad (53)$$

$$0 = -c_+ (\mu_1^2 + \mu_2^2) + \partial V_{4H} / \partial \chi_+^0, \quad (54)$$

$$0 = -c_- (\mu_1^2 - \mu_2^2) + \partial V_{4H} / \partial \chi_-^0. \quad (55)$$

Note that the derivative terms in (53)–(55) contain at least one of  $v_-$ ,  $c_+$  and  $c_-$ . Therefore, large values for  $\mu_1$  and  $\mu_2$  can be consistent with (53)–(55), only if (i)  $v_- = c_+ = c_- = 0$  and (ii)  $\mu_1^2 = \mu_2^2 + O(\text{VEV}^2)$  or (iii)  $\mu_1^2 = -\mu_2^2 + O(\text{VEV}^2)$ . Keeping this in mind, we next solve (51) and (52) to obtain

$$\mu_3^2 = \frac{v_+^2 (\mu_1^2 + \mu_2^2) + O(\text{VEV}^4)}{v_S^2}, \quad \mu_4^2 = -\frac{v_+ (\mu_1^2 + \mu_2^2) + O(\text{VEV}^3)}{\sqrt{2} v_S}. \quad (56)$$

Inserting (56) into the total Higgs potential  $V_T$ , we obtain

$$V_T = -(\mu_1^2 - \mu_2^2) H_-^\dagger H_- - \frac{\mu_1^2 + \mu_2^2}{2v_S^2} \left[ (v_S H_+^\dagger - v_+ H_S^\dagger)(v_S H_+ - v_+ H_S) + h.c. \right] + \dots \quad (57)$$

We see from (57) that the case (ii) can be ruled out, because in this case  $H_-$  can not obtain a large mass. We can also see from (57) that the case (iii) allows large values of the Higgs masses if  $|v_+/v_S| \gtrsim 40$ . However, (53) and (55) require that  $|v_-/v|, |c_-/v| \ll 1$ . Note that the derivative terms of (53) and (55) contain at least one of  $v_-$ ,  $c_-$ , which implies that  $v_- = c_- = 0$  to satisfy (53) and (55).  $c_+$  is nonvanishing in the case (iii). For the case (i) we obtain the same form of the leading potential  $V_T$ , but no restriction on the ratio  $v_+/v_S$ . In terms of VEVs, we have  $v_- = c_+ = c_- = 0$  for the case (i), and  $v_- = c_- = 0$  for (iii). These two types of VEVs are  $S'_2$  invariant VEVs (6). Both types of VEVs give rise to the general form of the fermion mass matrix (8).

Below we would like to consider only the case (i) ( $v_- = c_+ = c_- = 0$ ), and give the mass matrix  $\mathbf{m}_h^2$  of the neutral scalar Higgs bosons

$$h_-^0, h_L^0 = \sin \gamma h_+^0 + \cos \gamma h_S^0, h_H^0 = \cos \gamma h_+^0 - \sin \gamma h_S^0, \quad (58)$$

and the mass matrix  $\mathbf{m}_\chi^2$  the neutral pseudo scalar Higgs bosons

$$\chi_-^0, \chi_L^0 = \sin \gamma \chi_+^0 + \cos \gamma \chi_S^0, \chi_H^0 = \cos \gamma \chi_+^0 - \sin \gamma \chi_S^0, \quad (59)$$

are, respectively, given by

$$\mathbf{m}_h^2 = \begin{pmatrix} m_{h_-^0}^2 & 0 & 0 \\ 0 & m_{h_{22}}^2 & m_{h_{23}}^2 \\ 0 & m_{h_{23}}^2 & m_{h_{33}}^2 \end{pmatrix}, \mathbf{m}_\chi^2 = \begin{pmatrix} m_{\chi_-^0}^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_{\chi_H^0}^2 \end{pmatrix}, \quad (60)$$

where

$$m_{h_-^0}^2 = 2\mu_2^2 + \sqrt{2} \cot \gamma \mu_4^2 \simeq -(\mu_1^2 - \mu_2^2), \quad (61)$$

$$m_{h_{22}}^2 = v^2 [ 2(\lambda_1 + \lambda_3) \sin^4 \gamma + \frac{1}{2}(\lambda_5 + \lambda_6 + 2\lambda_7) \sin^2 2\gamma + 2\lambda_8 \cos^4 \gamma ], \quad (62)$$

$$m_{h_{23}}^2 = \frac{v^2}{2} \sin 2\gamma [ (\lambda_1 + \lambda_3)(1 - \cos 2\gamma) + (\lambda_5 + \lambda_6 + 2\lambda_7) \cos 2\gamma - 2\lambda_8 \cos^2 \gamma ], \quad (63)$$

$$m_{h_{33}}^2 = 2\sqrt{2}\mu_4^2 / \sin 2\gamma + \frac{v^2}{2}(\lambda_1 + \lambda_3 - \lambda_5 - \lambda_6 - 2\lambda_7 + \lambda_8) \sin^2 2\gamma, \quad (64)$$

$$m_{\chi_-^0}^2 = 2\mu_2^2 \sqrt{2} + \mu_4^2 \cot \gamma - 2(\lambda_2 + \lambda_3) v_+^2 - 2\lambda_7 v_S^2, \quad (65)$$

$$m_{\chi_H^0}^2 = 2\sqrt{2}\mu_4^2 / \sin 2\gamma - 2\lambda_7 v^2, \quad (66)$$

and we have introduced ( $v = (v_+^2 + v_S^2)^{1/2}$ )

$$\tan \gamma = \frac{v_+}{v_S}. \quad (67)$$

In (61)-(65), we have taken into account the higher order terms of (57) with  $\lambda_4 = \text{Im}(\lambda_7) = 0$ . [ $\lambda_4 = 0$  follows from the  $S'_2$  symmetry.] If  $\lambda_4$  and  $\text{Im}(\lambda_7)$  do not vanish, there is no local minimum for (i). As we can see from the mass matrices (60) with (61)-(65), the pseudo scalar boson (60),  $\chi_L$ , is the would-be Goldstone boson, and that except for  $h_L^0$  all the physical Higgs bosons can become heavy without large Higgs couplings  $\lambda$ 's. We also find from (58) and (67) that only  $h_L^0$  acquires VEV. Since only  $h_L^0$  acquires VEV, its coupling to the fermions is flavor diagonal, while the other physical neutral Higgs bosons have FCNC couplings. However,  $h_L^0$  still mixes with  $h_H^0$  because of the nonvanishing entry  $m_{h_{23}}^2$ . Therefore, we have to so

fine tune that  $m_{h23}^2$  vanishes. (Of course, the mixing is suppressed by  $v^2/\mu_4^2 \sim 6 \times 10^{-4}$  for  $\mu_4 \sim 10$  TeV.) In this limit,  $m_{h33}$  and  $m_{h22}$  are the masses of  $h_H^0$  and the lightest Higgs  $h_L^0$ , respectively.

#### IV. SOFT BREAKING WITHOUT SYMMETRY

Here we would like to investigate the full potential  $V_T = V_H + V_{SB}$  without any assumption on abelian discrete symmetries. The reason is that  $S'_2$  is not a symmetry of the theory; it can be a symmetry only in the Higgs potential. So, radiative corrections can induce finite non- $S'_2$ -invariant terms in the Higgs potential, for instance. Here we assume that all the soft masses (21) are present and that they are still real. We, however, do not allow an unnatural large hierarchy of the VEVs, in contrast to the previous sections. There are exactly nine nonequivalent possibilities that satisfy the phenomenological requirement (15):

$$A_1 : v_- = c_+ = c_- = 0; \quad A_2 : v_+ = v_- = c_- = 0; \quad (68)$$

$$B_1 : c_+ = c_- = 0; \quad B_2 : v_- = c_- = 0; \quad B_3 : v_- = c_+ = 0; \quad B_4 : v_+ = v_- = 0; \quad (69)$$

$$C_1 : c_- = 0; \quad C_2 : v_- = 0; \quad (70)$$

$$D : \text{none of them} = 0. \quad (71)$$

It will turn out that among these nine possibilities only two cases  $A_1$  and  $B_1$  satisfy the phenomenological constraint that all the Higgs bosons except one can be made heavy without running into the problem with triviality. Note that  $A_1$  and also  $B_2$  exhibit the  $S'_2$  invariant VEVs (6).

$A_1$  ( $v_- = c_+ = c_- = 0$ ):

We start with the case  $A_1$ . The first case  $A_1$  corresponds to the  $S'_2$  invariant VEVs (6). The nontrivial minimization conditions at  $v_- = c_+ = c_- = 0$  are given by

$$0 = -v_S \mu_3^2 - \sqrt{2} v_+ \mu_4^2 + \partial V_{4H} / \partial h_S^0, \quad (72)$$

$$0 = -v_+ (\mu_1^2 + \mu_2^2) - \sqrt{2} v_S \mu_4^2 + \partial V_{4H} / \partial h_+^0, \quad (73)$$

$$0 = -\sqrt{2} v_S \mu_6^2 - v_+ \mu_5^2 + \partial V_{4H} / \partial h_-^0 \quad (74)$$

$$0 = \partial V_{4H} / \partial \chi_+^0 = v_+ v_S (v_+ \text{Im}(\lambda_4) / 2\sqrt{2} + v_S \text{Im}(\lambda_7)), \quad (75)$$

$$0 = \partial V_{4H} / \partial \chi_-^0 = -v_+^2 v_S \text{Im}(\lambda_4) / 2\sqrt{2}. \quad (76)$$

(75) requires  $0 = \text{Im}(\lambda_7) + (v_+/2\sqrt{2}v_S)\text{Im}(\lambda_4)$ , and (76) requires  $\text{Im}(\lambda_4) = 0$ . So, we assume that  $\lambda_7$  and  $\lambda_4$  are real. (In the case of the  $S'_2$  invariant soft term (29),  $\lambda_4$  has to vanish for the  $S'_2$  VEVs (6) to correspond to a local minimum.) We then use (72)–(74) to express  $\mu_1^2, \mu_3^2$  and  $\mu_5^2$  in terms of VEVs:

$$\begin{aligned}\mu_1^2 &= -\mu_2^2 - \sqrt{2}\mu_4^2 \cot \gamma + O(\text{VEV}^2), \quad \mu_3^2 = -\sqrt{2}\mu_4^2 \tan \gamma + O(\text{VEV}^2), \\ \mu_5^2 &= -\sqrt{2}\mu_6^2 \cot \gamma + O(\text{VEV}^2),\end{aligned}\tag{77}$$

where  $\gamma$  is defined in (67). Inserting  $\mu$ 's of (77) into the total potential, we can compute the mass matrices and find

$$\mathbf{m}_h^2 \simeq \begin{pmatrix} 2\mu_2^2 + \sqrt{2}\mu_4^2 \cot \gamma & 0 & \sqrt{2}\mu_6^2 / \sin \gamma \\ 0 & 0 & 0 \\ \sqrt{2}\mu_6^2 / \sin \gamma & 0 & 2\sqrt{2}\mu_4^2 / \sin 2\gamma \end{pmatrix} + O(\text{VEV}^2) \simeq \mathbf{m}_\chi^2 \tag{78}$$

for the basis (58) and (59). Comparing these results with (60), we find that apart from the  $O(\text{VEV}^2)$  terms, the masses (78) reduce to those of the  $S'_2$  invariant case (60) as  $\mu_6^2$  (and hence  $\mu_5^2$  because of (77)) goes to zero. Therefore, the  $S'_2$  invariant local minimum exists in the full Higgs potential, if all the mass parameters are real.

$A_2$  ( $v_+ = v_- = 0$ ):

The five minimization conditions at  $v_+ = v_- = c_- = 0$  (which is of the  $S'_2$  invariant type (6)) are given by

$$0 = -v_S \mu_3^2 + \partial V_{4H} / \partial h_S^0, \tag{79}$$

$$0 = -\sqrt{2}v_S \mu_4^2 + \partial V_{4H} / \partial h_+^0, \tag{80}$$

$$0 = -\sqrt{2}v_S \mu_6^2 + \partial V_{4H} / \partial h_-^0. \tag{81}$$

$$0 = -c_+(\mu_1^2 + \mu_2^2) + \partial V_{4H} / \partial \chi_+^0, \tag{82}$$

$$0 = -c_+ \mu_5^2 + \partial V_{4H} / \partial \chi_-^0. \tag{83}$$

(79)–(83) imply <sup>4</sup> that  $\mu_3^2, (\mu_1^2 + \mu_2^2), \mu_4^2, \mu_5^2, \mu_6^2 \sim O(\text{VEV}^2)$ . Inserting  $\mu$ 's above into the total potential, we find

$$V_T = 2\mu_2^2 H_-^\dagger H_- + O(\text{VEV}^4). \tag{84}$$

---

<sup>4</sup> As announced, we do not allow an unnatural large hierarchy among the VEVs. If, for instance,  $|v_S/v| \ll 1$ , then  $\mu_3^2$  can be large thanks to the nonvanishing  $\lambda_4$ . In this case,  $H_S$  can become heavy.

So, only  $H_-$  can become heavy.

$B_1$  ( $c_+ = c_- = 0$ ):

The five minimization conditions at  $c_+ = c_- = 0$  are given by

$$0 = -v_S \mu_3^2 - \sqrt{2} v_+ \mu_4^2 - \sqrt{2} v_- \mu_6^2 + \partial V_{4H} / \partial h_S^0, \quad (85)$$

$$0 = -v_+ (\mu_1^2 + \mu_2^2) - \sqrt{2} v_S \mu_4^2 - v_- \mu_5^2 + \partial V_{4H} / \partial h_+^0, \quad (86)$$

$$0 = -v_+ \mu_5^2 - \sqrt{2} v_S \mu_6^2 - v_- (\mu_1^2 - \mu_2^2) + \partial V_{4H} / \partial h_-^0, \quad (87)$$

$$0 = -(v_+^2 + 2v_+ v_- - v_-^2) v_S \text{Im}(\lambda_4) / 2\sqrt{2} - v_+ v_S^2 \text{Im}(\lambda_7), \quad (88)$$

$$0 = -(v_+^2 - 2v_+ v_- - v_-^2) v_S \text{Im}(\lambda_4) / 2\sqrt{2} - v_- v_S^2 \text{Im}(\lambda_7). \quad (89)$$

(88) and (89) require  $\text{Im}(\lambda_7) = \text{Im}(\lambda_4) = 0$ . Solving (85), (86) and (87) to express  $\mu_1^2, \mu_3^2$  and  $\mu_6^2$  in terms of  $v_S, v_+$  and  $v_-$ , and inserting them into the total potential, we obtain:

$$\begin{aligned} V_T = & \left[ \{2\mu_2^2 v_-^2 + \mu_5^2 (v_-^3 / v_+ - v_+ v_-)\} / v_S^2 + \sqrt{2} \mu_4^2 (v_+ + v_-^2 / v_+) / v_S \right] H_S^\dagger H_S \\ & + [2\mu_4^2 (v_S / v_+) + \mu_5^2 (v_- / v_+)] H_+^\dagger H_+ \\ & + [2\mu_2^2 + \mu_5^2 (v_- / v_+) + \sqrt{2} \mu_4^2 (v_S / v_+)] H_-^\dagger H_- \\ & + \left[ -\sqrt{2} \mu_4^2 (v_- / v_+) - 2\mu_2^2 (v_- / v_S) + \mu_5^2 ((v_+ / v_S) - (v_-^2 / v_- v_S)) \right] (H_S^\dagger H_- + h.c) \\ & - \mu_5^2 (H_+^\dagger H_- + h.c) - \sqrt{2} \mu_4^2 (H_S^\dagger H_+ + h.c) + O(\text{VEV}^4). \end{aligned} \quad (90)$$

One can show that except for  $h_L = (v_S h_S^0 + v_+ h_+^0 + v_- h_-^0) / (v_+^2 + v_-^2 + v_S^2)^{1/2}$  all the physical Higgs bosons can become heavy. So, this case satisfies the phenomenological requirements.

$B_{2,3,4}, C_{1,2}, D$ :

We have performed similar analyses for the rest of the cases, and found that none of  $B_{2,3,4}$ ,  $C_{1,2}$  and  $D$  cases satisfy our requirement (if we do not allow a large hierarchy among the VEVs).

## V. THE PAKVASA-SUGAWARA VACUUM

The Pakvasa-Sugawara (PS) VEVs [1] are given by

$$v_- = c_+ = 0, \quad (91)$$

which is nothing but the case  $B_3$  given in (69). As we mentioned, the  $S_3$  invariant potential (9) does meet the requirement that except for one neutral physical Higgs boson, all the physical bosons can become heavy. On the other hand, the PS VEVs (91) are the most economic VEVs in the case of a spontaneous CP violation; only one phase which should be determined in the Higgs sector enters into the Yukawa sector. Here we would like to analyze the most general case with complex soft masses in contrast to the previous sections. The minimization conditions are:

$$0 = -v_S \mu_3^2 - \sqrt{2} v_+ \text{Re}(\mu_4^2) + \sqrt{2} c_- \text{Im}(\mu_6^2) + \partial V_{4H} / \partial h_S^0, \quad (92)$$

$$0 = -v_+ (\mu_1^2 + \mu_2^2) - \sqrt{2} v_S \text{Re}(\mu_4^2) + c_- \text{Im}(\mu_5^2) + \partial V_{4H} / \partial h_+^0, \quad (93)$$

$$0 = -\sqrt{2} v_S \text{Re}(\mu_6^2) - v_+ \text{Re}(\mu_5^2) + \partial V_{4H} / \partial h_-^0, \quad (94)$$

$$0 = \sqrt{2} v_S \text{Im}(\mu_4^2) - c_- \text{Re}(\mu_5^2) + \partial V_{4H} / \partial \chi_+^0, \quad (95)$$

$$0 = \sqrt{2} v_S \text{Im}(\mu_6^2) + v_+ \text{Im}(\mu_5^2) - c_- (\mu_1^2 - \mu_2^2) + \partial V_{4H} / \partial \chi_-^0. \quad (96)$$

We solve (92)-(96) to express  $\mu_1^2, \mu_3^2, \text{Im}(\mu_4^2), \text{Re}\mu_5^2$  and  $\text{Im}(\mu_5^2)$  in terms of VEVs. We find that in the leading order, they are given by

$$\begin{aligned} \mu_1^2 &= \left( \mu_2^2 v_+^2 + \sqrt{2} \text{Re}(\mu_4^2) v_+ v_S + \sqrt{2} \text{Im}(\mu_6^2) v_S c_- + \mu_2^2 c_-^2 \right) / (c_-^2 - v_+^2) \\ &\quad + O(\text{VEV}^2), \\ \mu_3^2 &= \sqrt{2} (-\text{Re}(\mu_4^2) (v_+ / v_S) + \text{Im}(\mu_6^2) (c_- / v_S)) + O(\text{VEV}^2), \\ \text{Im}(\mu_4^2) &= -\text{Re}(\mu_6^2) (c_- / v_+) + O(\text{VEV}^2), \quad \text{Re}(\mu_5^2) = -\sqrt{2} \text{Re}(\mu_6^2) (v_S / v_+) + O(\text{VEV}^2), \\ \text{Im}(\mu_5^2) &= \left( \sqrt{2} \text{Re}(\mu_4^2) v_S c_- + \sqrt{2} \text{Im}(\mu_6^2) v_+ v_S + 2\mu_2^2 v_+ c_- \right) / (c_-^2 - v_+^2) \\ &\quad + O(\text{VEV}^2). \end{aligned} \quad (97)$$

Inserting these mass parameters into the full potential, we have verified numerically that except for one neutral physical Higgs boson, all the physical bosons can become heavy. In the limit, in which the imaginary parts of  $\mu_4^2, \mu_5^2, \mu_6^2, \lambda_4$  and  $\lambda_7$  vanish, the Pakvasa-Sugawara VEVs reduce to the  $S'_2$  invariant VEVs (6), as we can see also from

$$c_- \rightarrow \left( -4\text{Im}(\mu_4^2) + \text{Im}(\lambda_4) v_+^2 + 2\sqrt{2} \text{Im}(\lambda_7) v_+ v_S \right) (v_+ / 4\text{Re}(\mu_6^2)) + \dots, \quad (98)$$

where  $\dots$  stands for higher orders in the limit.

## VI. SUPERSYMMETRIC EXTENSION

As in the case of the minimal supersymmetric standard model (MSSM), we introduce two  $S_3$  doublet Higgs superfields,  $H_i^U, H_i^D (i = 1, 2)$ , and two  $S_3$  singlet Higgs superfields,  $H_S^U, H_S^D$  [10, 11]. The same R-parity is assigned to these fields as in the MSSM. Then the most general renormalizable  $S_3$  invariant superpotential is given by

$$W_H = \mu_1 H_i^U H_i^D + \mu_3 H_S^U H_S^D. \quad (99)$$

The  $S_3$  invariant soft scalar mass terms are [10, 11]:

$$\begin{aligned} \mathcal{L}_S = & -m_{H_1^U}^2 (|\hat{H}_1^U|^2 + |\hat{H}_2^U|^2) - m_{H_1^D}^2 (|\hat{H}_1^D|^2 + |\hat{H}_2^D|^2) \\ & -m_{H_S^U}^2 (|\hat{H}_S^U|^2) - m_{H_S^D}^2 (|\hat{H}_S^D|^2), \end{aligned} \quad (100)$$

and the  $S_3$  invariant B terms are:

$$\mathcal{L}_B = B_1(\hat{H}_1^U \hat{H}_1^D + \hat{H}_2^U \hat{H}_2^D) + B_3(\hat{H}_S^U \hat{H}_S^D) + h.c., \quad (101)$$

where hatted fields are scalar components. Given the superpotential (99) along with the  $S_3$  invariant soft supersymmetry breaking (SSB) sector (100) and (101), we can now write down the scalar potential. For simplicity we assume that only the neutral scalar components of the Higgs supermultiplets acquire VEVs. The relevant part of the scalar potential is then given by

$$\begin{aligned} V = & (|\mu_1|^2 + m_{H_1^U}^2)(|\hat{H}_1^{0U}|^2 + |\hat{H}_2^{0U}|^2) + (|\mu_1|^2 + m_{H_1^D}^2)(|\hat{H}_1^{0D}|^2 + |\hat{H}_2^{0D}|^2) \\ & + (|\mu_3|^2 + m_{H_S^U}^2)(|\hat{H}_S^{0U}|^2) + (|\mu_3|^2 + m_{H_S^D}^2)(|\hat{H}_S^{0D}|^2) \\ & + \frac{1}{8}(\frac{3}{5}g_1^2 + g_2^2)(|\hat{H}_1^{0U}|^2 + |\hat{H}_2^{0U}|^2 + |\hat{H}_S^{0U}|^2 - |\hat{H}_1^{0D}|^2 - |\hat{H}_2^{0D}|^2 - |\hat{H}_S^{0D}|^2)^2 \\ & - [B_1(\hat{H}_1^{0U} \hat{H}_1^{0D} + \hat{H}_2^{0U} \hat{H}_2^{0D}) + B_3(\hat{H}_S^{0U} \hat{H}_S^{0D}) + h.c.], \end{aligned} \quad (102)$$

where  $g_{1,2}$  are the gauge coupling constants for the  $U(1)_Y$  and  $SU(2)_L$  gauge groups. As one can see easily, the scalar potential  $V$  (102) has a continuous global symmetry  $SU(2) \times U(1)$  in addition to the local  $SU(2)_L \times U(1)_Y$ . As a result, there will be a number of pseudo Goldstone bosons that are phenomenologically unacceptable. This is a consequence of  $S_3$  symmetry. Therefore, we would like to break  $S_3$  symmetry explicitly. As in the non-supersymmetric case, we would like to break it as soft as possible to preserve predictions from  $S_3$  symmetry,



while breaking the global  $SU(2) \times U(1)$  symmetry completely. There is a unique choice for that: Since the softest terms have the canonical dimension two, the soft  $S_3$  breaking should be in the SSB sector. As for the soft scalar masses, we have an important consequence (100) from  $S_3$  symmetry that they are diagonal in generations. Since we would like to preserve this, the only choice is to introduce the soft  $S_3$  breaking terms in the B sector [11]. Moreover, looking at the  $S_3$  invariant scalar potential  $V$  (102), we observe that it has again an abelian discrete symmetry

$$S'_2 : H_1^{U,D} \leftrightarrow H_2^{U,D}, \quad (103)$$

which is the same as (7). We assume that the soft  $S_3$  breaking terms respect this discrete symmetry (103), and add the following soft  $S_3$  breaking Lagrangian:

$$\mathcal{L}_{S_3 B} = B_4(\hat{H}_1^U \hat{H}_2^D + \hat{H}_2^U \hat{H}_1^D) + B_5 \hat{H}_S^U (\hat{H}_1^D + \hat{H}_2^D) + B_6 \hat{H}_S^D (\hat{H}_1^U + \hat{H}_2^U) + h.c. \quad (104)$$

In the following discussions, we assume that all the B parameters are real. The resulting scalar potential can be analyzed, and one finds that a local minimum respecting  $S'_2$  symmetry, i.e.

$$\begin{aligned} \langle \hat{H}_1^{0U} \rangle &= \langle \hat{H}_2^{0U} \rangle = v_U/2 \neq 0, \quad \langle \hat{H}_1^{0D} \rangle = \langle \hat{H}_2^{0D} \rangle = v_D/2 \neq 0, \\ \langle \hat{H}_S^{0U} \rangle &= v_{SU}/\sqrt{2} \neq 0, \quad \langle \hat{H}_S^{0D} \rangle = v_{SD}/\sqrt{2} \neq 0, \end{aligned} \quad (105)$$

can occur. To see this, we write down the minimization conditions in this case, which can be uniquely solved:

$$(|\mu_1^2|^2 + m_{H_1^U}^2) = (B_1 v_{SD} + B_4 v_{SD} + \sqrt{2} B_6 v_{SD})/v_U + O(\text{VEV}^2), \quad (106)$$

$$(|\mu_3^2|^2 + m_{H_S^U}^2) = (B_3 v_{SD} + \sqrt{2} B_5 v_D)/v_{SU} + O(\text{VEV}^2), \quad (107)$$

$$(|\mu_1^2|^2 + m_{H_1^D}^2) = (B_1 v_U + B_4 v_U + \sqrt{2} B_5 v_{SU})/v_D + O(\text{VEV}^2), \quad (108)$$

$$(|\mu_3^2|^2 + m_{H_S^D}^2) = (B_3 v_{SU} + \sqrt{2} B_6 v_U)/v_{SD} + O(\text{VEV}^2), \quad (109)$$

Inserting these solutions into the scalar potential (102) with (104), we obtain the mass matrices for the Higgs fields. As in the non-supersymmetric case, we redefine the Higgs fields as

$$\hat{H}_{\pm}^{D,U} = \frac{1}{\sqrt{2}}(\hat{H}_1^{D,U} \pm \hat{H}_2^{D,U}). \quad (110)$$

Then the mass matrices can be written as

$$\mathbf{M}_-^2 = \begin{pmatrix} ((B_1 + B_4)v_D + \sqrt{2}B_6v_{SD})/v_U & -B_1 + B_4 \\ -B_1 + B_4 & ((B_1 + B_4)v_U + \sqrt{2}B_5v_{SU})/v_D \end{pmatrix} + O(\text{VEV}^2) \quad (111)$$

for the  $(\hat{H}_-^U, (\hat{H}_-^D)^\dagger)$  basis, and

$$\mathbf{M}^2 = \begin{pmatrix} M_{US} & 0 & -B_3 & -\sqrt{2}B_5 \\ 0 & M_{U+} & -\sqrt{2}B_6 & -B_1 - B_4 \\ -B_3 & -\sqrt{2}B_6 & M_{DS} & 0 \\ -\sqrt{2}B_5 & -B_1 - B_4 & 0 & M_{D+} \end{pmatrix} + O(\text{VEV}^2) \quad (112)$$

for the  $(\hat{H}_S^U, \hat{H}_+^U, (\hat{H}_S^D)^\dagger, (\hat{H}_+^D)^\dagger)$  basis, where

$$M_{US} = (B_3v_{SD} + \sqrt{2}B_5v_D)/v_{SU}, \quad M_{U+} = ((B_1 + B_4)v_D + \sqrt{2}B_6v_{SD})/v_U, \quad (113)$$

$$M_{DS} = (B_3v_{SU} + \sqrt{2}B_6v_U)/v_{SD}, \quad M_{D+} = ((B_1 + B_4)v_U + \sqrt{2}B_5v_{SU})/v_D. \quad (114)$$

From the mass matrices (111) and (112), we find that the lightest physical Higgs boson, the MSSM Higgs boson, can be written as a linear combination

$$h_{\text{MSSM}} = (v_D \hat{H}_+^{0D} + v_{SD} \hat{H}_S^{0D} + v_U \hat{H}_+^{0U} + v_{SU} \hat{H}_S^{0U})/v, \quad (115)$$

where  $v = (v_U^2 + v_{SU}^2 + v_D^2 + v_{SD}^2)^{1/2} \simeq 246$  GeV, and its mass is approximately given by

$$m_h \simeq ((3/5)g_1^2 + g_2^2)(v_U^2 + v_{SU}^2 - v_D^2 - v_{SD}^2)^2/v \quad (116)$$

for  $\mu'^2 s, B's \gg v^2$ . It can be shown that the masses of the other physical Higgs bosons can be made arbitrary heavy. From (116) we see that the tree-level upper bound for  $m_h$  is exactly the same as in the MSSM.

Because of the very nature of the SSB terms, the explicit breaking of  $S_3$  in the B sector (104) does not propagate to the other sector. Moreover, although the superpotential (99) and the corresponding trilinear couplings do not respect  $S'_2$  symmetry (103), they can not generate  $S'_2$  violating infinite B terms because they can generate only  $S_3$  invariant terms, which are however automatically  $S'_2$  invariant.

## VII. CONCLUSIONS

We recall that our investigations have been carried out under the two phenomenological conditions (14) and (15). Below we would like to summarize our conclusions.

1: The  $S_3$  invariant Higgs potential (9) does not satisfy the phenomenological requirement that

“except one neutral physical Higgs boson all the physical Higgs bosons can become heavy  $\gtrsim 10$  TeV without having a problem with triviality. ”

That is, for a phenomenological viable model we have to break  $S_3$  explicitly if we do not introduce further Higgs fields.

2: Among the real nonequivalent soft  $S_3$  breaking masses (28), (29) and (31) that can be characterized according to discrete symmetries, only the  $S'_2$  invariant case (29) with the  $S'_2$  invariant VEVs (6) can satisfy the phenomenological requirement of 1.

3: Even for the most general quartic Higgs potential with the most general real  $S_3$  breaking masses (21), the  $S'_2$  invariant VEVs (6) can correspond to a local minimum and satisfy the phenomenological requirement of 1.

4: The Pakvasa-Sugawara VEVs (91) can be a local minimum in the case of the most general quartic Higgs potential with the most general complex  $S_3$  breaking masses and can satisfy the phenomenological requirement of 1.

5: In a minimal supersymmetric extension with the  $S'_2$  invariant, real soft  $S_3$  breaking masses in the B sector, the phenomenological requirement of 1 can be satisfied with the  $S'_2$  invariant VEVs (105), where the other B parameters are also assumed to be real. These B terms violate supersymmetry as well as  $S_3$  softly. This possibility to introduce  $S_2$  violating soft terms in the B sector only is consistent with renormalizability. The lower bound of the lightest Higgs boson is the same as in the MSSM.

It is a very difficult task to test the Higgs sector experimentally. However, as we see from (57) and (60), in the case of the  $S'_2$  invariant soft breaking with the  $S'_2$  invariant VEVs (6), there are basically only two masses  $m_{h_H}$  and  $m_{h_-}$  for four neutral and two charged heavy Higgs bosons. This may be experimentally tested, because their couplings to the fermions are fixed [8, 9].

## Acknowledgments

One of us (JK) would like to thank Manuel Drees and Manfred Lindner for useful discussions, and Walter Grimus for useful discussions and his hospitality at the Universität Wien. This work is supported by the Grants-in-Aid for Scientific Research from the Japan Society for the Promotion of Science (No. 13135210).

- 
- [1] S. Pakvasa and H. Sugawara, Phys. Lett. **B73**, 61 (1978).
  - [2] S. Pakvasa and H. Sugawara, Phys. Lett. **B82**, 105 (1979).
  - [3] Y. Yamanaka, H. Sugawara and S. Pakvasa, Phys. Rev. **D25**, 1895 (1982); Erratum **D29**, 2135 (1984).
  - [4] T. Brown, N. Desphande, S. Pakvasa and H. Sugawara, Phys. Lett. **B141**, 95 (1984).
  - [5] H. Harari, H. Haut and T. Weyers, Phys. Lett. **B78**, 459 (1978).
  - [6] Y. Koide, Lett. Nuovo Cimento **34**, 201 (1982); Phys. Lett. **B120**, 161 (1983); Phys. Rev. **D28**, 252 (1983).
  - [7] H. Fritzsch and Z.Z. Xing, Prog. Part. Nucl. Phys. **45**, 1 (2000).
  - [8] J. Kubo, A. Mondragón, M. Mondragón and E. Rodríguez-Jáuregui, Prog. Theor. Phys. **109**, 795 (2003).
  - [9] J. Kubo, Phys. Lett. **B578**, 156 (2004).
  - [10] T. Kobayashi, J. Kubo and H. Terao, Phys. Lett. **B568**, 83 (2003).
  - [11] Ki-Y. Choi, Y. Kajiyama, J. Kubo and H.M. Lee, hep-ph/402026.
  - [12] S-L. Chen and E. Ma, hep-ph/040315.
  - [13] C.I. Low and R.R. Volkas, Phys. Rev. **D68**, 033007 (2003); C.I. Low, hep-ph/0404017.
  - [14] W. Grimus and L. Lavoura, Phys. Lett. **B572**, 76 (2003).
  - [15] E. Ma and G. Rajasekaran, Phys. Rev. **D64**, 113012 (2001); E. Ma, Mod. Phys. Lett. **A17**, 627 (2002); 2361 (2002) .
  - [16] K.S. Babu, E. Ma and J.W.F. Valle, Phys. Lett. **B552**, 207 (2003).
  - [17] K.S. Babu, T. Kobayashi and J. Kubo, Phys. Rev. **D67**, 075018 (2003).
  - [18] M. Hirsch, J.C. Romao, S. Skadhauge, J.W.F. Valle and A. Villanova del Moral, hep-ph/0312265.

- [19] W. Grimus and L. Lavoura, JHEP **0107**, 045 (2001); Acta Phys. Polon. **B32**, 3719 (2001).
- [20] E. Ma and G. Rajasekaran, Phys. Rev. **D68**, 071302 (2003); E. Ma, hep-ph/0308282; hep-ph/0401025.
- [21] Y. Koide, Phys. Rev. **D60**, 077301 (1999).
- [22] Y. Koide, H. Nishiura, K. Matsuda, T. Kikuchi and T. Fukuyama, Phys. Rev. **D66**, 093006 (2002).
- [23] T. Ohlsson and G. Seidl, Phys. Lett. **B537**, 95 (2002); Nucl. Phys. **B643**, 247 (2002).
- [24] T. Kitabayashi and M. Yasue, Phys. Rev. **D67**, 015006 (2003).
- [25] W. Grimus and L. Lavoura, hep-ph/0305309; hep-ph/0312330.
- [26] M. Lüscher and P. Weisz, Nucl. Phys. **B290**, 25 (1987); Nucl. Phys. **B295**, 65 (1988); *ibid.* **B318**, 705 (1989).
- [27] J. Kubo, Talk given at the DESY Theory Workshop, Sept. 23-26, 2003, Hamburg, Germany.
- [28] H. Okada, Master thesis, Kanazawa University (2004); F. Sakamaki, Master thesis, Kanazawa University (2004).
- [29] K.S. Babu and M.S. Barr, Phys. Lett. **B525**, 289 (2002).